Computation of the structure of module categories using FD Applet

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I will give an overview of *FD Applet* [1], an easy-to-use application that I am developing. It can compute various properties related to modules over (mainly representation-finite special biserial) algebras. For example, FD Applet can of course compute the Auslander-Reiten quiver of a given (representation-finite special biserial) algebra. In addition, this can compute the complete lists (and the numbers) of

- (Miyashita / Wakamatsu / τ- / classical / cluster-)tilting modules,
- 2-term simple-minded collections,
- τ-tilting-theoretical objects such as support τ-tilting modules, τ-rigid modules, semibricks, torsion classes, wide subcategories,
- cotorsion pairs,
- resolving subcategories,
- subcategories closed under images and extensions / images, (co)kernels, and extensions,

and so on. Moreover, this can compute various modules / subcategories from a given module / subcategory:

- from a module to the smallest torsion class containing it,
- from a support τ-tilting module to the corresponding semibrick,
- from a semibrick to the corresponding 2-term simple-minded collection,
- from a torsion class to the corresponding wide subcategory,
- from a given subcategory to the Ext-projective objects in it,

and so on. This can also compute some quivers like the quiver of support τ-tilting modules with the brick labeling and the Hasse quiver of various subcategories.

In this talk, I will demonstrate the use of FD Applet, and then explain the algorithm used in FD Applet. More precisely, we review the structure of modules over special biserial algebras and see that we only need *string combinatorics* to calculate the above things (we do not even need linear algebras). Then I will explain how specific objects like torsion classes and resolving subcategories can be computed.

In addition, I would like to talk about possible applications or impacts of FD Applet. One such application is my preprint about orthogonal modules [2], and possibly many enumerative research and experiments can be done using FD Applet. I would like to appreciate any feature requests from the audience, and also any contributions or developers to FD Applet.

If time permits, I will also talk about the combination of computer and mathematics in another direction: the proof assistant system *Lean*: we can formalize (that is, teach to computer) various theories of modern mathematics including algebras and category theory.

References

[1] H. Enomoto, *FD Applet*, https://haruhisa-enomoto.github.io/fd-applet/, Online Version: https://fd-applet.dt.r.appspot.com/.

[2] H. Enomoto, Maximal self-orthogonal modules and a new generalization of tilting modules, arXiv:2301.13498.

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