Classifications of Exact Structures and Cohen-Macaulay-finite Iwanaga-Gorenstein Algebras

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Outline

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Categories of Finite Type $=$ Algebras

k: a field.

Proposition

There exists a bijection between:

1. *Hom-finite k-categories $\mathcal{E}$ of finite type (:$\Leftrightarrow$ categories with finitely many indecomposables).*
2. *Finite-dimensional k-algebras Γ (we call Γ an Auslander algebra of $\mathcal{E}$).*

Idea

1. Categorical properties of $\mathcal{E}$ and
2. Homological behavior of its Auslander algebra Γ

should be related!

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Theorem (Auslander correspondence)

TFAE for the above $\mathcal{E}$ and Γ.

(1) $\mathcal{E}$ is abelian. (Categorical!)

(2) $\mathrm{gl.dim}\,\Gamma \leq 2 \leq \mathrm{dom.dim}\,\Gamma$ (Homological!)

What Corresponds to Exact Structures?

(1)		(2)
{ Cats of fin-type + Exact str. on it }	$\overset{?}{\longleftrightarrow}$	{ Algebras + some info }

$\mathcal{E}$ + exact str. $\longmapsto$ Γ + ?

Our Aim

is to Classify Exact Structures on a given category

using its Auslander algebra.

Outline

Exact Category

$0 \to X \xrightarrow{f} Y \xrightarrow{g} Z \to 0$ is a kernel-cokernel pair in $\mathcal{E}$
$:\Leftrightarrow f = \ker g$ and $g = \operatorname{coker} f$.

Definition (Quillen 1973)

An exact category consists of a pair $(\mathcal{E}, F)$, where

- $\mathcal{E}$ is an additive category, and
- F is a class of ker-coker pairs in $\mathcal{E}$ (called F-exact)

satisfying some conditions.

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Example

Extension-closed subcategory of abelian categories
(e.g. torsion class, CM rep-theory, $\cdots$)

Auslander Algebras of Categories of Finite Type

From now on, fix a field k and

- Algebra = finite-dimensional k-algebra.
- Category = Krull-Schmidt Hom-finite k-category.
- $\mathcal{E}$: a category of finite type.

Definition

An Auslander algebra Γ of $\mathcal{E}$ is defined by $\Gamma := \mathrm{End}_{\mathcal{E}}(G)$, where G is the additive generator of $\mathcal{E}$ ($\mathcal{E} = \mathrm{add}\, G$).

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$$\{\text{ Cats of fin-type }\} \xleftrightarrow{\text{1-1}} \{\text{Algebras}\}$$

$$\mathcal{E} \longmapsto \Gamma := \mathrm{End}_{\mathcal{E}}(G)$$

$$\mathcal{E} := \mathrm{proj}\,\Gamma \longleftarrow\!\shortmid\; \Gamma$$

Projectivization

$\Gamma := \mathrm{End}_{\mathcal{E}}(G)$: the Auslander algebra of $\mathcal{E}$.

Proposition (Auslander's "Projectivization")

We have (anti-)equivalences :

$$P_{(-)} := \mathcal{E}(G,-) : \mathcal{E} \xrightarrow{\sim} \mathsf{proj}\,\Gamma,$$
$$P^{(-)} := \mathcal{E}(-,G) : \mathcal{E} \xrightarrow{\sim} \mathsf{proj}\,\Gamma^{\mathrm{op}},$$

which satisfies

$$\mathrm{Hom}_{\Gamma}(P_{(-)},\Gamma) = P^{(-)},$$
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Proposition (Auslander's "Projectivization")

We have (anti-)equivalences ,something like Yoneda emb.:

$$P_{(-)} := \mathcal{E}(G,-) : \mathcal{E} \xrightarrow{\sim} \operatorname{proj}\Gamma, \quad P_X \quad \text{“} = \text{”} \quad \mathcal{E}(-,X)$$
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Ker-Coker pair in $\mathcal{E}$ in terms of Γ-module

$\mathcal{E}$: cat of fin. type, Γ: its Auslander algebra.

Proposition

$0 \to X \xrightarrow{f} Y \xrightarrow{g} Z \to 0$ *is a ker-coker pair in* $\mathcal{E}$

$\Leftrightarrow$

1. *The following is exact:*

$$0 \to \mathcal{E}(-, X) \xrightarrow{f\circ} \mathcal{E}(-, Y) \xrightarrow{g\circ} \mathcal{E}(-, Z).$$

2. *The following is exact:*

$$0 \to \mathcal{E}(Z, -) \xrightarrow{\circ g} \mathcal{E}(Y, -) \xrightarrow{\circ f} \mathcal{E}(X, -).$$

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$0 \to X \xrightarrow{f} Y \xrightarrow{g} Z \to 0$ *is a ker-coker pair in* $\mathcal{E}$

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$$0 \to P_X \xrightarrow{f\circ} P_Y \xrightarrow{g\circ} P_Z \qquad .$$

2. *The following is exact in* $\operatorname{mod}\Gamma^{\mathrm{op}}$

$$0 \to P^Z \xrightarrow{\circ g} P^Y \xrightarrow{\circ f} P^X \qquad .$$

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$0 \to X \xrightarrow{f} Y \xrightarrow{g} Z \to 0$ *is a ker-coker pair in* $\mathcal{E}$

$\Leftrightarrow$ *for* $M := \operatorname{Coker}(P_Y \to P_Z)$ *in* $\operatorname{mod}\Gamma$,

1. *The following is exact in* $\operatorname{mod}\Gamma$ $\rightsquigarrow$ pd $M_\Gamma \leq 2$

$$0 \to P_X \xrightarrow{f\circ} P_Y \xrightarrow{g\circ} P_Z \to M \to 0.$$

2. *The following is exact in* $\operatorname{mod}\Gamma^{\mathrm{op}}$ $\rightsquigarrow$ $\operatorname{Ext}^{0,1}_\Gamma(M, \Gamma) = 0$.

$$0 \to P^Z \xrightarrow{\circ g} P^Y \xrightarrow{\circ f} P^X \to \operatorname{Ext}^2_\Gamma(M, \Gamma) \to 0.$$

Ker-Cok pairs in $\mathcal{E} \leftrightarrow$ Objects in $\mathcal{C}_2(\Gamma)$

Definition

The subcat $\mathcal{C}_2(\Gamma) \subset \operatorname{mod}\Gamma$ consists of Γ-modules M satisfying

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Exact str. on $\mathcal{E}$ $\longleftrightarrow$???

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Ker-cok pair in $\mathcal{E}$ ⟷ Obj in $\mathcal{C}_2(\Gamma)$

Class of ker-cok pairs ⟷ Subcat of $\mathcal{C}_2(\Gamma)$

$\cup$ $\cup$

Exact str. on $\mathcal{E}$ ⟷ modules supported at 2-regular simples

2-Regular Condition

Definition

A simple Γ-module S is called 2-regular $:\Leftrightarrow$

1. $S \in \mathcal{C}_2(\Gamma)$, that is, $\operatorname{pd} S_\Gamma = 2$ and $\operatorname{Ext}_\Gamma^{0,1}(S, \Gamma) = 0$.
2. $\operatorname{Ext}_\Gamma^2(S, \Gamma)$ is a simple Γ^{op}-module.

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2-regular simple Γ-mod correspond to AR ker-coker pairs in $\mathcal{E}$:

$0 \to X \to Y \to Z \to 0$: AR ker-cok pair in $\mathcal{E}$	$\longleftrightarrow$	$0 \to P_X \to P_Y \to P_Z \to S \to 0$ 2-reg. simple Γ-mod S

AR Quivers and Main Result

$\mathcal{E}$: cat. of fin. type, Γ: its Auslander algebra.

Definition

The AR quiver $Q(\mathcal{E})$ of $\mathcal{E}$ is the translation quiver defined by:

- Quiver = the usual quiver of $\mathcal{E}$ (or Γ)
- $X \dashleftarrow Z$ if $\exists$ an AR ker-cok pair $0 \to X \to Y \to Z \to 0$ in $\mathcal{E}$.

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Theorem [E]

There exists a bijection between the following classes.

1. Exact structures F on $\mathcal{E}$.
2. Sets S of 2-regular simple Γ-modules.
3. Sets $\mathbb{A}$ of dotted arrows in $Q(\mathcal{E})$.

Outline

$\nexists$ Auslander Correspondence for CM-fin IG Alg?

Definition

- Λ is Iwanaga-Gorenstein if id Λ_Λ, id$_\Lambda \Lambda < \infty$.
- $X \in \operatorname{mod} \Lambda$ is Cohen-Macaulay if $\operatorname{Ext}_\Lambda^{>0}(X, \Lambda) = 0$.
- CM Λ: the cat. of CM Λ-modules. $\rightsquigarrow$ an exact cat!
- Λ: CM-finite if CM Λ is of fin. type.

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$$\{\text{ CM-fin algebras }\} \xrightarrow{\text{CM}} \{\text{ Cats of fin-type }\} \xleftrightarrow{\text{1-1}} \{\text{Algebras}\}$$

$$\Lambda \longmapsto \text{CM}\,\Lambda \longmapsto \Gamma$$

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Important point

We CANNOT recover Λ from its CM category,

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Important point

We CANNOT recover Λ from its CM category,
but CAN if its exact str. is given!

Characterizing CM categories of IG algebras

$$\left\{\begin{array}{c}\text{CM-finite}\\ \text{IG-alg. } \Lambda\end{array}\right\} \xhookrightarrow{\text{CM}} \left\{\begin{array}{c}\text{Exact cats } \mathcal{E}\\ \text{of finite type}\end{array}\right\} \xleftrightarrow[\checkmark]{\text{1-1}} \left\{\begin{array}{c}\text{Alg. } \Gamma + \text{sets of}\\ \text{dotted arrows}\end{array}\right\}$$

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$\mathcal{E}$: exact cat. of fin. type, Γ: its Auslander algebra.

Proposition (Kalck-Iyama-Wemyss-Yan,E)

$\mathcal{E} \simeq \mathrm{CM}\,\Lambda$ *as exact cats for some IG algebra* $\Lambda \Leftrightarrow$

1. $\mathrm{gl.dim}\,\Gamma < \infty$.
2. *Projective objects in* $\mathcal{E}$ = *Injective objects in* $\mathcal{E}$ *(equivalently,* $\mathcal{E}$ *is Frobenius).*

“Classification" of CM-finite IG alg.

Corollary

There exists a bijection between the following.

1. CM-finite Iwanaga-Gorenstein algebras Λ.
2. Pairs $(\Gamma, \mathbb{A})$, where Γ is an algebra with $\mathrm{gl.dim}\,\Gamma < \infty$ and $\mathbb{A}$ is a set of dotted arrows of $Q(\Gamma)$ which is union of stable τ-orbits

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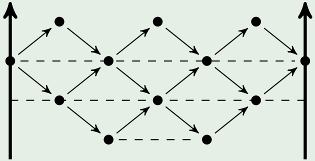
Remark

- Γ parametrizes possible additive str. of CM category.
- $\mathbb{A}$ parametrizes possible Frobenius exact str. on that cat., or equivalently, possible IG alg. whose CM cats are that cat.

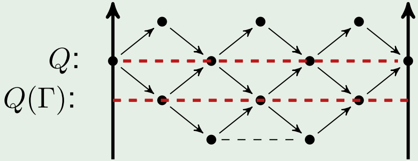
Example

Q:



$\Gamma := kQ/$(commutativity and zero relation)
(two vertical arrows are identified).

Example



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$\Rightarrow$ the above is $Q(\Gamma)$. Thus $\exists$ 2 stable τ-orbits.
$\rightsquigarrow$ We obtain $2^2 = 4$ CM-fin IG algebras Λ.

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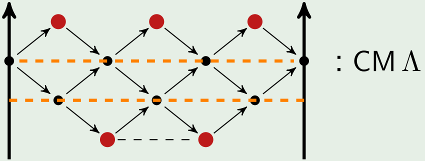
$Q(\Gamma)$:



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$\mathbb{A}$: Orange Dotted Arrows.
Corresponding CM-finite IG Λ is the End of Red vertices, projective object in this exact structure.

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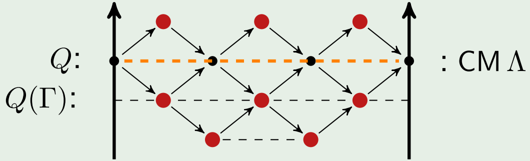

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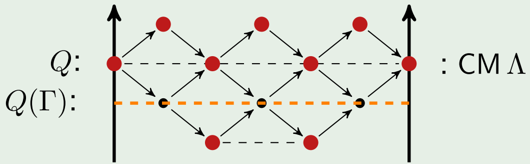

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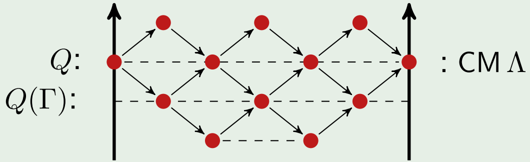

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NON-Example

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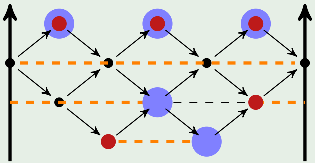

If $\mathbb{A}$: Orange, then in the corresponding exact str,

- Red: proj. objects,
- Blue: injective objects.

$\rightsquigarrow$ Proj $\neq$ Inj. (not Frobenius)
(This exact cat. is ${}^{\perp}U$ for some cotilting Λ-module U)

Example

$\mathcal{E} :=$ mod $k[\bullet \leftarrow \bullet \leftarrow \bullet]$.

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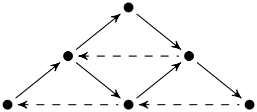

$\exists$ 3 dotted arrow, hence

$\exists$ $2^3 = 8$ exact str. on $\mathcal{E}$

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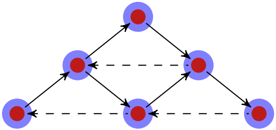

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Orange arrows are chosen.

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No arrows $\leftrightarrow$ trivial exact str. of $\mathcal{E}$ (the smallest one).

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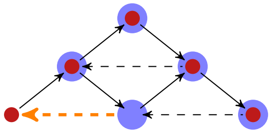

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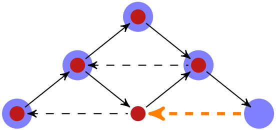

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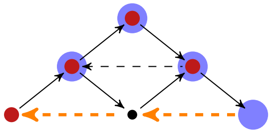

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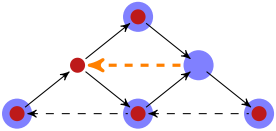

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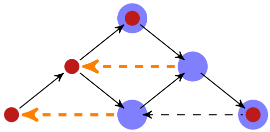

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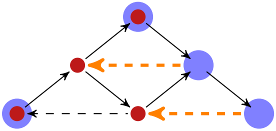

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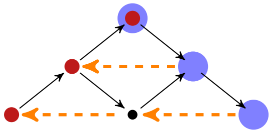
$\mathcal{E} := \text{mod}\, k[\bullet \leftarrow \bullet \leftarrow \bullet]$.

$Q(\mathcal{E})$:



$\exists$ 3 dotted arrow, hence

$\exists\ 2^3 = 8$ exact str. on $\mathcal{E}$

Orange arrows are chosen.

- Red: proj. objects,
- Blue: injective objects.

All arrows $\leftrightarrow$ usual exact str. of $\mathcal{E}$ (the largest one).

Thank you for your attention!