

Computing various objects of an algebra from the poset of torsion classes (Demo)

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About SageMath

Installation

Input the poset of torsion classes

Enjoy!

About SageMath

What is SageMath?

A free open-source mathematics software system

Why SageMath?

- SageMath provides a framework for **finite posets**, **finite lattices**, **simplicial complexes**, and so on.
- We can **construct** them and **compute** various things, e.g. the set of join-irreducibles in the lattice.
- SageMath can check whether two objects are **isomorphic**.

I've developed a SageMath program `tors_lattice.py`

1. Input the poset of torsion classes, then
2. This program can construct
 - the lattice of wide subcategories,
 - the simplicial complex $\Delta(\Lambda)$,etc.

Installation

Step 1: Install SageMath

- Install SageMath (ver ≥ 9.0) on your computer:
Download from the official site
<https://www.sagemath.org/index.html>
and just install it!
- From now on, we'll use SageMath Notebook

Another option:

You can use CoCalc without installing sage

Step 2: Download my program and load it

- Download my file from
https://haruhisa-enomoto.github.io/files/tors_lattice.py
- Create a SageMath notebook in the same directory as you download this file.
- Execute `load("tors_lattice.py")`

Then you can use it!

Input the poset of torsion classes

Input your poset

First, input the poset of torsion classes of your algebra in SageMath, and name it `poset` for example.

Ways to input posets

1. For path algebras and preprojective algebras of Dynkin type, SageMath already has it! (later)
2. If you have a Hasse diagram, then you can input it manually.
3. You can import it from Jan Geuenich's [String Applet](#) using my other program, next slide.

String Applet to SageMath converter

String Applet can calculate the poset of torsion classes of any representation-finite special biserial algebra.

You can import it in SageMath using my **converter**.

1. Input your algebra in **String Applet**, and show its st -Tilting quiver.
2. Export a **latex** file (e.g. **data.tex**) in your working directory.

3. Download

<https://haruhisa-enomoto.github.io/files/converter.py>

in your working directory

4. Load it in your notebook by `load("converter.py")`
5. Execute `poset = Poset(SAtoSage("data.tex"))`

Enjoy!



What's next?

Now you have your `poset`. Then execute

```
tors = TorsLattice(poset)
```

Once you have done it, you can construct various things, e.g.

- `tors.wide_lattice()`
the lattice of wide subcategories
- `tors.ice_lattice()`
the lattice of ICE-closed subcategories
- `tors.heart_poset()`
the poset of torsion hearts
- `tors.s_tau_tilt_complex()`
the simplicial complex $\Delta(\Lambda)$ of τ -tilting pairs

See [Manual](#) for the list of all things you can do.

Q: Dynkin quiver, Π_Q : its preprojective algebra

algebra	tors(-)	wide(-)	$\Delta(-)$
kQ	Cambrian lattice	Non-crossing partition	Cluster cpx
Π_Q	Weyl grp with weak order	shard intersection order	(???)

All these objects are already in SageMath!

Let's check the above table, and guess (???)!

Q: Dynkin quiver, Π_Q : its preprojective algebra

algebra	tors(-)	wide(-)	$\Delta(-)$
kQ	Cambrian lattice	Non-crossing partition	Cluster cpx
Π_Q	Weyl grp with weak order	shard intersection order	Coxeter cpx

Red are conjectured by SageMath experiment (then proved).

Demo: lattice properties for wide Λ

There're lots of properties SageMath can check for lattices.

Conjecture (some have been confirmed)

If Λ is τ -tilting finite, then wide Λ is:

- ranked (graded), with its rank function given by the number of simples
- Rank-symmetric
- Relatively complemented
- (strongly) Sperner

Conjecture of ICE-closed subcategories

Sakai's Conjecture (not true...)

The number of Hasse arrows in $\text{ICE } \Lambda$ starting at \mathcal{C} is equal to the number of indecomposable Ext-projectives in \mathcal{C} .

True for hereditary and Nakayama algebras.

Find a counterexample!

Links

- [The SageMath notebook used in this demo](#)
- [SageMath codes in my website](#)
- On SageMath
 - [SageMath tutorial](#)
 - [Finite Coxeter groups](#)
 - [Finite posets](#)
 - [Finite lattices and semilattices](#)
- The lattice of torsion classes in SageMath
 - [Download](#)
 - [Manual](#)
- String Applet to SageMath converter
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