MAXIMAL SELF-ORTHOGONAL MODULES AND A NEW GENERALIZATION OF TILTING MODULES

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This talk is based on the paper [En]. Let Λ be a finite-dimensional k-algebra over a field k, and let $\mathsf{mod}\,\Lambda$ denote the category of finitely generated Λ-modules. We say that $T \in \mathsf{mod}\,\Lambda$ is *self-orthogonal* if $\mathrm{Ext}^i_\Lambda(T,T) = 0$ for all $i > 0$. In this talk, I will present my results on self-orthogonal modules.

We consider a generalization of the progenerator Λ. This Λ is self-orthogonal and *generates* $\mathsf{mod}\,\Lambda$ in the sense that every object in $\mathsf{mod}\,\Lambda$ admits a surjection from Λ^m for some m. There is a notion of *tilting modules* that generalizes Λ, and it satisfies a similar property: if T is tilting, then consider the subcategory $T^\perp := \{X \in \mathsf{mod}\,\Lambda \mid \mathrm{Ext}^{>0}_\Lambda(T,X) = 0\}$ of $\mathsf{mod}\,\Lambda$. Then, T generates $T^\perp$ by [AR].

Building on this property, we define a *projectively Wakamatsu tilting module* to be a self-orthogonal module T that generates $T^\perp$. This is a subclass of *Wakamatsu tilting modules* introduced by Wakamatsu [Wa].

Our main result is as follows: suppose Λ is *representation-finite*, meaning that there are only finitely many non-isomorphic indecomposable Λ-modules. Let $|T|$ denote the number of non-isomorphic indecomposable summands of a module $T \in \mathsf{mod}\,\Lambda$. Then, any self-orthogonal module can be completed into a projectively Wakamatsu tilting module, and the following statements are equivalent for $T \in \mathsf{mod}\,\Lambda$:

(1) T is projectively Wakamatsu tilting.
(2) T is Wakamatsu tilting.
(3) T is self-orthogonal with $|T| = |\Lambda|$.
(4) T is maximal self-orthogonal, meaning that T is self-orthogonal and if $T \oplus M$ is self-orthogonal, then M is a direct summand of T^m for some m.

If time permits, I will also discuss several open conjectures related to self-orthogonal modules.

References

[AR] M. Auslander, I. Reiten, *Applications of contravariantly finite subcategories*, Adv. Math. 86 (1991), no. 1, 111–152.

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[Wa] T. Wakamatsu, *Stable equivalence for self-injective algebras and a generalization of tilting modules*, J. Algebra 134 (1990), no. 2, 298–325.

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